Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA. Sem- IST Regular / Remedial Examination January/ February 2011

Subject code: 610003

Subject Name: Discrete Mathematics for Computer Science

Date: 31 /01 /2011 **Time: 10.30 am – 01.00 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Define a Boolean algebra. Show that lattice $<P(A), \cup, \cap>$ is a Boolean algebra, where A = {a, b, c} and P(A) denotes its power set. What are the operations of meet and join in it? Draw the Hasse diagram of this Boolean algebra. **07**

(b) For the poset $<\{\{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,3,4\}, \{2,3,4\}\}, \subseteq>$ Draw the Hasse diagram and find : **07**

1) maximal elements and minimal elements
2) Greatest element and least element, if exists
3) Lower bounds of {1,3,4} and {2,3,4}
4) Upper bounds of {2,4} and {3,4}

Q.2 (a) Answer the following.

1) Prove that if "All men are mortal." and "Socrates is a man." Then "Socrates is a mortal." by using theory of Inference. **04**
2) Determine the truth value of each statement given below. The domain of discourse is the set of real numbers. Justify your answers. **03**
   i) For every x, $x^2 > x$
   ii) For some x, $x^2 > x$
   iii) For every x, if $x > 1$ then $x^2 > x$.

(b) Do the following.

1) Give an example of **04**
   i) A bounded lattice which is complemented but not distributive.
   ii) A bounded lattice which is distributive but not complemented.
   iii) A bounded lattice which is neither distributive nor complemented.
   iv) A bounded lattice which is both distributive and complemented.

2) Two equivalence relations R and S are given by their relation matrices MR and MS. Show that RoS is not an equivalence relation. **03**

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad M_S = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

OR

(b) Define an equivalence relation. Prove that the relation "congruence modulo m" given by ≡ {<x, y> / x-y is divisible by m} over the positive integer is an equivalence relation. Also draw the relation graph for this relation using m=5 over the set x={1,2,3,……,10}. **07**

Q.3 (a) Answer the following.

1) Define isomorphic lattices. Draw the Hasse diagrams of lattices **03**
   i) $(S_4 \times S_{25}, D)$ ii) (S_{36}, D)
   check whether these lattices are isomorphic?

2) Define complemented lattice. Which of two lattices <Sn, D> for n=30 and n=45 are complemented? Draw Hasse Diagram of these lattices. Are these lattices distributive? Justify your answer. **04**

(b) Answer the following.

1) Define Sub-Boolean Algebra. Find all Sub-Boolean Algebra of $<S_{100},D>$. **03**

2) For a Boolean Algebra $<B, *, \oplus, ', 0, 1>$ prove that **04**

$(a \oplus b') *(b \oplus c')* (c \oplus a') = (a' \oplus b) * (b' \oplus c) * (c' \oplus a)$

OR

Q.3 **(a)** Answer the following.

1) Use the Quine McClusky method to simplify the SOP expansion, **05**

$F(a,b,c,d) = \Sigma\ (0, 2, 4, 6, 8, 10, 12, 14)$

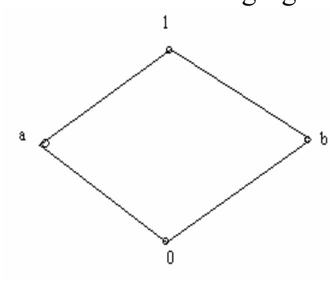
And draw the circuit diagram of the minimized function.

2) In any Boolean algebra, show that **02**

$a \le b \Rightarrow a \oplus (b * c) = b * (a \oplus c)$

(b) Answer the following.

1) Given an expression $\alpha(x1, x2, x3)$ defined to be $\sum(0, 3, 5, 7)$, determine the value of $\alpha(a, b, 1)$, where $a, b, 1 \in B$ and $<B, *, \oplus, ‘, 0, 1>$ is a Boolean algebra given in the following figure. **04**



2) Let $(B, *, \oplus, ', 0, 1)$ be a Boolean algebra prove the following : **03**

$a = b \Leftrightarrow (a * b') \oplus (a' * b) = 0$

Q.4 **(a)** Define “Group” “normal subgroup” “Group homomorphism” of a group. Determine all the proper subgroups of the symmetric group $<S_3, \Diamond>$ given in the table below. Is this group normal? Justify your answer. **07**

◊	P1	P2	P3	P4	P5	P6
P1	P1	P2	P3	P4	P5	P6
P2	P2	P1	P5	P6	P3	P4
P3	P3	P6	P1	P5	P4	P2
P4	P4	P5	P6	P1	P2	P3
P5	P5	P4	P2	P3	P6	P1
P6	P6	P3	P4	P2	P1	P5

(b) Define isomorphic groups. Prove that groups $<Z_5^*, X_5>$ and $<Z_4, +_4>$ are isomorphic, where $Z_5^* = Z_5 - [0]$ **07**

OR

Q.4 **(a)** Define cyclic group. Show that cyclic group is abelian but converse is not true. **07**
Is $<z_5, +_5>$ a cyclic group? If so, find its generators.

(b) Define subgroup of a group, left coset of a subgroup $<H, *>$ in the group $<G, *>$. Find left cosets of $\{[0], [3]\}$ in the group $<Z_6, +_6>$. **07**

Q.5 **(a)** Give an abstract definition of graph. When are two simple graphs said to be isomorphic? Give an example of two simple digraphs having 4 nodes and 4 edges which are not isomorphic. **07**

(b) When a simple digraph is said to be weakly connected, unilaterally connected and strongly connected? Define weak, unilateral and strong components. Write the Strong, unilateral and weak components for the diagraph given in **fig-1**. **07**

OR

Q.5 **(a)** Define nodebase of a simple diagraph. Find the reachability set of all nodes for the diagraph given in **fig-2**. Also find the nodebase for it. **07**

(b) Give three other representations of tree expressed by **07**
(v0(v1(v2)(v3)(v4))(v5(v6)(v7)(v8)(v9))(v10(v11)(v12)))
Obtain binary tree corresponding to it.

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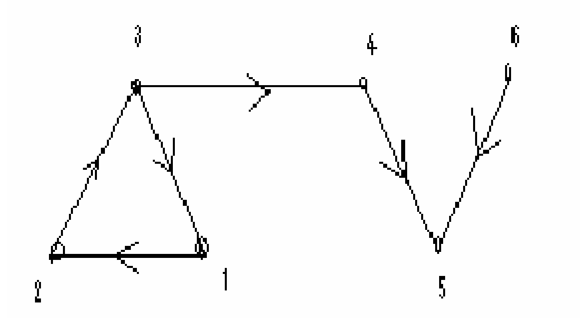

fig-1

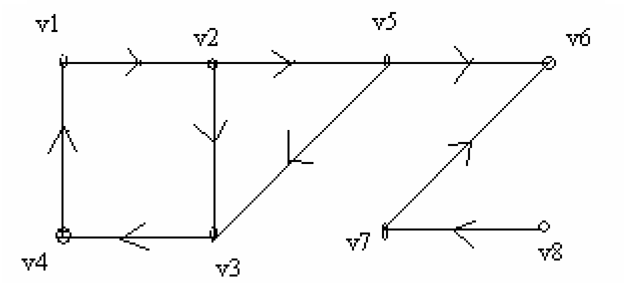

fig-2

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Seat No.: _________ Enrolment No._______________

GUJARAT TECHNOLOGICAL UNIVERSITY
MCA SEM-I Examination- Jan.-2012

Subject code: 610003 **Date: 04/01/2012**
Subject Name: Discrete Mathematics for Computer Science (DMCS)
Time: 10.30 am-1.00 pm **Total marks: 70**

Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q:1 Do as directed.

(a) 1. Let A= {1, 2, 3} then show that the inclusion relation $\subseteq$ on A is a partial ordering. **03**

2. Define Direct Product and draw the Hasse diagrams of <S,D>, <L,D> and <S X L, D> for S = {1,3,6} and L = {1,2,4}. **03**

3. State the Lagrange's theorem. **01**

(b) 1. Briefly discuss the following terms: **02**
   i) modus pones
   ii) hypothetical syllogisms

2. Explain Isomorphic graphs with example. **04**

3. Write negation of following statement: **01**
   This flower is not beautiful.

Q.2 Do as directed.

(a) 1. Let A = {1,2,3} and $R=\{(x,y)/x<y\}$ then find M_R and construct the digraph of R. **03**

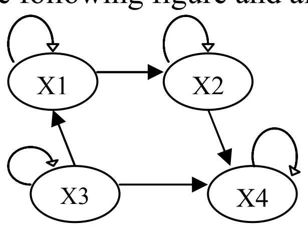
2. Use a truth table to determine whether the following argument form is valid. **02**

$p \rightarrow q$
q
Therefore p

3. Explain abelian group with example. **02**

(b) Do as directed.

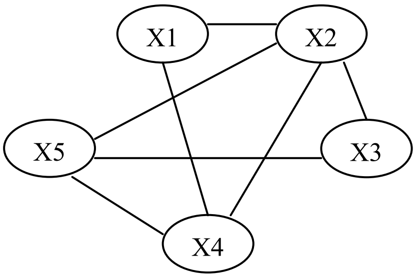
1. Determine the properties of the relations given by the graph shown in the following figure and also write the corresponding relation matrix. **02**



2. Let S={1,2,3,4} and consider the following collections of subsets of S. Find which subsets are partitions and which subsets are covering of S. **02**

A = {{1,2},{2,3,4}}
B = {{1,2},{3,4}}
C = {{1},{1,2}}

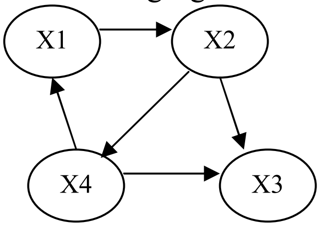
3. Find out maximum compatibility blocks of following digraph and write its relation matrix. **02**



4. Define Poset with example. **01**

OR

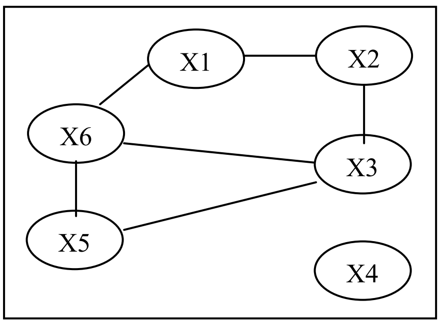
(b) 1. Determine the properties of the relations given by the graph shown in the following figure and also write the corresponding relation matrix. **02**



2. Let S={a,b,c} and consider the following collections of subsets of S. Find which subsets are partitions and which subsets are covering of S. **02**

A = {{a,b},{c}}
B = {{a,b},{b,c}}
C = {{a},{a,b}}

3. Find out maximum compatibility blocks of following digraph and write its relation matrix. **02**



4. Define Equivalence Relation with example. **01**

Q.3 Do as directed.

(a) 1. If R and S are equivalence relations on the set X, prove that $R \cap S$ is an equivalence relation. **03**

2. Let R and S be two relations on a set of positive integers I, **02**

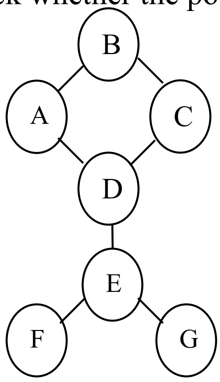
$R = \{<x,2x>/x \in I\}$
$S = \{<x,7x>/x \in I\}$

Then find R 0 S and R 0 R 0 R.

3. Let X = {2,3,6,12,24,36} and the relation ≤ be such that x ≤ y if x divides y. Draw the Hasse Diagram of <x, ≤>. **02**

(b) 1. Given a relation R on the set A = {1,2,3,4,5,6} and the equivalence classes are {1,3,6}, {2} and {4,5} then represent them by using Vector Representation Method. **04**

2. Check whether the poset shown below represents lattice or not? **03**



OR

Q.3 Do as directed.

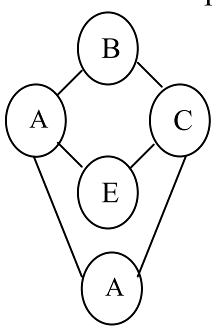
(a) 1. Let R be a relation on N such that aRb if and only if a|b (a divides b) in N. Check whether R is an equivalence relation or not? **03**

2. Let R and S be two relations on a set of positive integers I, **02**
R = {<x,3x>/x∈I}
S = {<x,4x>/x∈I}
Then find R 0 R and R 0 S 0 R.

3. Let S = {2,4,5,10,15,20} and the relation ≤ is the divisibility relation. Draw the Hasse diagram of <S, ≤>. **02**

(b) 1. Show that in a lattice, a ≤ b ⇔ a * b = a ⇔ a ⊕ b = b, where <L, ≤> be a lattice in which * and ⊕ denote the operations of meet and join respectively, for all a,b ∈ L. **04**

2. Check whether the poset shown below represents lattice or not? **03**



Q.4 Do as directed.

(a) 1. Show that < S_{30},*,⊕ > and <P(A), ∩,∪> are isomorphic lattices for A = {a,b,c}. **02**

2. Consider the set B = {0, 1}. Show that <B,*, ⊕,’,0,1> is a Boolean algebra. **02**

3. Write an algorithm COVER which determines the set of prime implicants by using the Quine-McCluskey procedure. **03**

(b) 1. Prove or disprove: If x is an irrational number, then x^2 is irrational. **02**

2. Give an indirect proof to show that if n is an even integer, then n is odd. **02**

3. Prove the following theorem using direct proof method. **03**
For all integer x, if x is odd, then x^2 is odd.

OR

Q:4 Do as directed.

(a) 1. Consider lattice < S_{10}, *, ⊕ >, where S_{10} = {1,2,5,10}. Find the complements of the elements of S_{10}. **02**

2. Write the following Boolean expression in an equivalence sum-of-products canonical form in three variables x1,x2 and x3. **02**
x1 ⊕ x2.

3. Discuss Karnaugh map with example. **03**

(b) 1. Prove without constructing the truth table that p Λ (~q → ~p) → q is a tautology **02**

2. Explain biimplication with example **02**

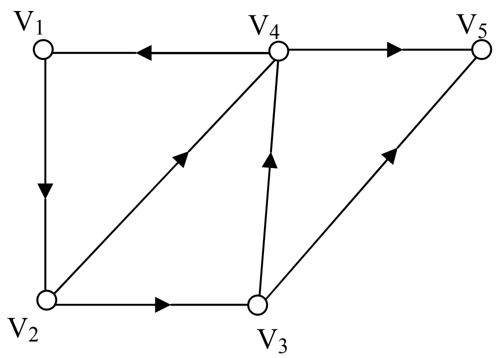
3. Let P(x,y) denote the sentence: 2x + y = 1. What are the truth values of ∀x∃yP(x,y), ∀x∀yP(x,y), and ∃x∃yP(x,y), where the domain of x,y is the set of all integers? **03**

Q.5 **(a)** 1. Explain path and circuit with example. **04**

2. Test the validity of the logical consequence given below: **03**
All birds can fly.
A sparrow is a bird.
Therefore, a sparrow can fly.

(b) 1. Define the following terms: **02**
i) subgroup ii) group homomorphism

2. Find all the indegrees and outdegrees of the nodes of the graph given in following figure. **05**
Give all the elementary cycles of this graph. List all the nodes which are reachable from another node of the diagraph.



OR

Q.5 **(a)** 1. Explain Adjacency matrix of graph G with example. **04**

2. What is a group? List at least two properties of group. **03**

(b) 1. Show that every cyclic group of order n is isomorphic to the group (Z_n, +n) **02**

2. What is ring in group theory? **02**

3. Write a short note on a directed tree **03**

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Seat No.: _____ Enrolment No ._______

GUJARAT TECHNOLOGICAL UNIVERSITY

M. C. A. Semester - IST Examination –July- 2011

Subject code: 610003

Subject Name: Discrete Mathematics for Computer Science

Date:08/07/2011 **Time: 02:30 pm – 05:00 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Without constructing truth table prove that $\sim(p \vee q \vee r) \equiv \sim p \wedge \sim q \wedge \sim r$ **02**

(ii) Give an example of a relation which is neither reflexive nor irreflexive. **02**

(iii) Define group with example. Give an example of non-abelian group. **03**

(b) (i) If predicate $R(x){:}\, x\ is\ a\ prime\ integer$ then write a statement in English corresponding to the following statements: $R(10)$ and $\exists x\, R(x)$. **02**

(ii) Define Boolean expression. **02**

(iii) Define isomorphic graphs and give its example. **03**

Q.2 **(a)** (i) Define statement formula. **02**

(ii) Define Lattice as a partially ordered set with example. **02**

(iii) Define sum-of-product canonical form. Write Boolean expression $x_1 * x_2$ in an equivalent sum-of-products canonical form in three variables x_1, x_2 and x_3. **03**

(b) Explain contradiction method and using it prove that $\sqrt{50}$ is an irrational number. **07**

OR

(b) For an integer x prove that the following statements are equivalent: **07**

p: x is divisible by 10.

q: x is divisible by 2 and 5.

r: x is an even number and x is divisible by 5.

Q.3 **(a)** Show that the operations of meet and join on a lattice are commutative, associative and idempotent. **07**

(b) Use the Karnaugh map representation to find a minimal sum-of-products expression of $f(a,b,c,d) = \sum(0,1,2,3,13,15)$. **07**

OR

Q.3 **(a)** Define Distributive lattice. Give an example of non-distributive lattice. Also give an example to verify that every chain is a distributive lattice. **07**

(b) Define Boolean function and Symmetric Boolean expression. Determine $a'bcd + a'c' + b'c'd' + ad'$ is symmetric or not. **07**

Q.4 **(a)** State and prove DeMorgan's Law for Boolean Algebra. **07**

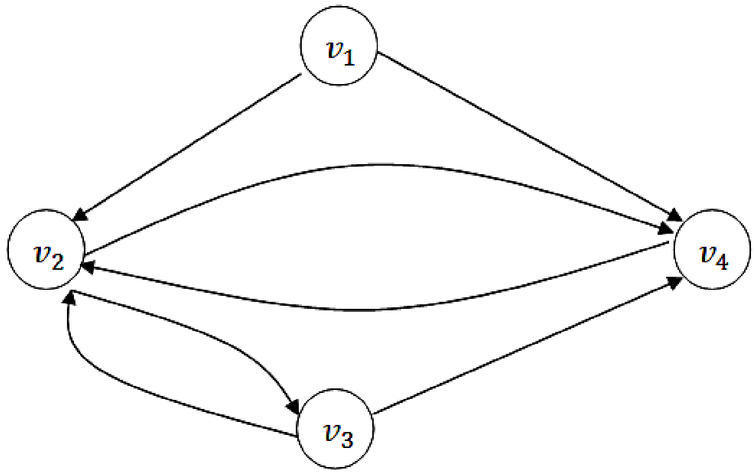
(b) Find all the subgroups of S_4 generated by the permutations **07**

$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}$ and $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$.

OR

Q.4 **(a)** Describe the application of Boolean algebra to Relational Database with example. **07**

Q.4 **(b)** Obtain the adjacency matrix A of the digraph given below. Also find the elementary paths of lengths 1 and 2 from v_1 to v_4. **07**



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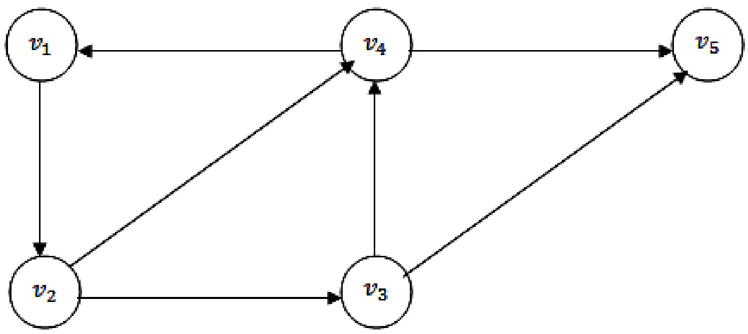
Q.5 **(a)** Let $\langle G, * \rangle$ be a finite cyclic group generated by an element $a \in G$. If G is of order n then prove that n is the least positive integer for which $a^n = e$. **07**

(b) Find the strong components of the following diagraph. Also find its unilateral and weak components. **07**



OR

Q.5 **(a)** Define left coset and right coset. Find the left cosets of $\{[0],[3]\}$ in the group $\langle Z_6, +_6 \rangle$. **07**

(b) Define a directed tree. Show by means of an example that a simple digraph in which exactly one node has indegree 0 and every other node has indegree 1 is not necessarily a directed tree. **07**

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Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA Sem-I Examination January 2010

Subject code: 610003

Subject Name: Discreet Mathematics for Computer Science

Date: 21 / 01/ 2010 **Time: 12.00 -2.30 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define “Boolean expression”. Show that **07**
$[a * (b' \oplus c)]' * [b' \oplus (a * c')']' = a * b * c'$

(b) Define “Symmetric Boolean expression”. Determine whether the following functions are symmetric or not: **07**
(i) $a'bc' + a'c'd + a'bcd + abc'd$
(ii) $abc' + ab'c + a'bc + ab'c' + a'bc' + a'b'c$

Q.2 **(a)** Define “Universal quantifier” and “Existential quantifier”. **07**
(i) Express the following sentences into logical expression using First Order Predicate Logic:
“All lines are fierce”
“Some student in this class has got university rank”
(ii) Show the following implication without constructing the truth tables first and thereafter show it through the truth tables.
$(P \to Q) \to Q \Rightarrow (P \vee Q)$

(b) Define equivalence relation. **07**
Let Z be the set of integers and R be the relation called “Congruence modulo 5” defined by
$R = \{\langle x,y\rangle \mid x \in Z \wedge y \in Z \wedge (x - y) \text{ is divisible by } 5\}$
Show that R is an equivalence relation. Determine the equivalence classes generated by the elements of Z.

OR

(b) Define “compatibility relation” and “maximal compatibility block”. Let the compatibility relation on a set $(x_1, x_2, \ldots, x_6\}$ be given by the matrix **07**

x_2	1				
x_3	1	1			
x_4	0	0	1		
x_5	0	0	1	1	
x_6	1	0	1	0	1
	x_1	x_2	x_3	x_4	x_5

Draw the graphs and find the maximal compatibility blocks of the relation.

Q.3 **(a)** Define "Composite relation" and "Converse of a relation". **07**
Given the relation matrix M_R of a relation R on the set {a, b, c}, find the relation matrices of ~R (Converse of a R),
R^2 = R o R and R o ~R.

$$M_R = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

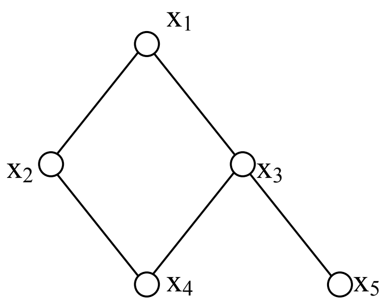
(b) Prove the following Boolean Identities: **04**
(i) $a \oplus (a \oplus b')' = a \oplus b$
(ii) $a * (a * b')' = a * b$

(c) Find the six left cosets of H = {p_1, p_5, p_6} in the group $\langle S_3, * \rangle$, given in the following table: **03**

*	p_1	p_2	p_3	p_4	p_5	p_6
p_1	p_1	p_2	p_3	p_4	p_5	p_6
p_2	p_2	p_1	p_5	p_6	p_3	p_4
p_3	p_3	p_6	p_1	p_5	p_4	p_2
p_4	p_4	p_5	p_6	p_1	p_2	p_3
p_5	p_5	p_4	p_2	p_3	p_6	p_1
p_6	p_6	p_3	p_4	p_2	p_1	p_5

OR

Q.3 **(a)** (i) Define "Partial order relation" and "Chain". **07**
(ii) The following figure gives the Hesse diagram of a partially ordered set <P, R>, where P = {x_1, x_2, x_3, x_4, x_5}.



Find which of the following are true:
x_1 R x_2, x_4 R x_1, x1 R x_1, and x_2 R x_5. Find the upper and lower bounds of {x_2, x_3, x_4}, {x_3, x_4, x_5}, {x_1, x_2, x_3}

(b) Show that **04**
(i) $a + 0 = a$
(ii) $a + 1 = a'$
(iii) $a + a = 0$
(iv) $a + a' = 1$
where $a + b = (a * b') \oplus (a' * \text{b})$

(c) Show that $\langle S_3, * \rangle$ as given in the above table [i.e. Q.3(c) main part] is a group. [Note: Only one non-trivial example to show associativity will be sufficient. **03**

Q.4 **(a)** Define "Group", "Order of a group", and "Abelian Group". **07**

For $P = \{ p_1, p_2, \ldots p_5\}$ and $Q = \{ q_1, q_2, \ldots, q_5\}$ explain why (P, *) and <Q, Δ> are not groups. The operations * and Δ are given in the following table:

*	p_1	p_2	p_3	p_4	p_5	Δ	q_1	q_2	q_3	q_4	q_5
p_1	p_1	p_2	p_3	p_4	p_5	q_1	q_4	q_1	q_5	q_3	q_2
p_2	p_2	p_1	p_4	p_5	p_3	q_2	q_3	q_5	q_2	q_1	q_4
p_3	p_3	p_5	p_1	p_2	p_4	q_3	q_1	q_2	q_3	q_4	q_5
p_4	p_4	p_3	p_5	p_1	p_2	q_4	q_2	q_4	q_1	q_5	q_3
p_5	p_5	p_4	p_2	p_3	p_4	q_5	q_5	q_3	q_4	q_2	q_1

(b) Define "Lattice as an Algebraic System", "Direct Product of Lattices" and "Complete Lattice". **07**

Let the sets $S_0, S_1, \ldots, S_7$ be given by

$S_0 = \{a, b, c, d, e, f\}$, $S_1 = \{a, b, c, d, e\}$, $S_2 = \{a, b, c, e, f\}$, $S_3 = \{a, b, c, e\}$, $S_4 = \{a, b, c\}$, $S_5 = \{a, b\}$, $S_6 = \{a, c\}$,
$S_7 = \{a\}$

Draw the diagram of $<L, \subseteq>$,
where $L = \{S_0, S_1, S_2, \ldots, S_7\}$

OR

Q.4 **(a)** Define "Subgroup", "Group Isomorphism", and "Kernel of the homomorphism". **07**

Show that the groups <G, *> and <S, Δ> given by the following table are isomorphic.

*	p_1	p_2	p_3	p_4	Δ	q_1	q_2	q_3	q_4
p_1	p_1	p_2	p_3	p_4	q_1	q_3	q_4	q_1	q_2
p_2	p_2	p_1	p_4	p_3	q_2	q_4	q_3	q_2	q_1
p_3	p_3	p_4	p_1	p_2	q_3	q_1	q_2	q_3	q_4
p_4	p_4	p_3	p_2	p_1	q_4	q_2	q_1	q_4	q_3

(b) Define "Sub Lattice", "Lattice homomorphism" and "Distributive Lattice". **07**

Find all the sub lattices of the lattice $<S_n, D)$ for n = 12, i.e. the lattice of divisors of 12 in which the partial ordering relation D means "division".

Q.5 **(a)** Define Directed Graph, Cycle, Path, In degree, Binary Tree **05**

(b) Can we say that any square Boolean Matrix will definitely represent a directed graph? What does a 4x4 unit matrix represent? **05**

Draw the graph corresponding to the following Boolean Matrix:

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

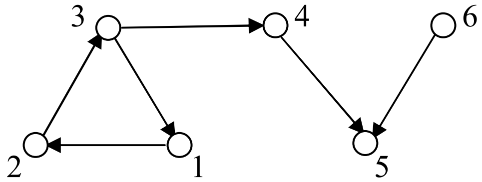
How many (>=0) cycles does this graph have? Write down all the cycles. Which single edge is to be deleted to convert this graph into a cyclic graph?

(c) From the adjacency matrix of a simple digraph, how will you determine whether it is a directed tree? If it is a directed tree, how will you determine its root and terminal nodes? **04**

OR

Q.5 **(a)** Define Graph, Loop, Out Degree, Tree, Node Base **05**

(b) Find the strong components of the digraph given below: **05**



Also find its unilateral components. Give brief valid reasons/justification for your answer.

(c) Define complete binary tree. Show through two examples with $n_t = 7$ and $n_t = 8$ of complete binary trees that the total number of edges is given by $2(n_t - 1)$, where n_t is the number of terminal nodes. **04**

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Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA Sem-I Remedial Examination April 2010

Subject code: 610003

Subject Name: Discrete Mathematics For Computer Science

Date: 06 / 04 / 2010 **Time: 12.00 noon – 02.30 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Answer the following:

(i) Express the following using predicates, quantifiers, and logical connectives. Also verify the validity of the consequence. **04**

Everyone who graduates gets a job.
Ram is graduated.
Therefore, Ram got a job.

(ii) Prove by contradiction that $\sqrt{2}$ is an irrational number. **03**

(b) Draw Hasse Diagram of the poset $\langle\{2,3,5,6,9,15,24,45\},D\rangle$. Find **07**

(i) Maximal and Minimal elements
(ii) Greatest and Least members, if exist.
(iii) Upper bound of $\{9,15\}$ and l.u.b. of $\{9,15\}$, if exist.
(iv) Lower bound of $\{15,24\}$ and g.l.b. of $\{15,24\}$, if exist.

Q.2 **(a)** When a poset said to be lattice? Explain. Is every poset a lattice? Justify. Is the poset $\langle\{\varnothing,\{p\},\{q\},\{p,q,r\}\},\ \subseteq\rangle$ lattice? **07**

(b) Show that the lattice $\langle S_n,\ D\rangle$ for n = 100 is isomorphic to the direct product of lattices for n = 4 and n = 25. **07**

OR

(b) With proper justification give an example of **07**

(1) A bounded lattice which is complemented but not distributive.
(2) A bounded lattice which is distributive but not complemented.
(3) A bounded lattice which is neither distributive nor complemented.
(4) A bounded lattice which is both distributive and complemented.

Q.3 **(a)** Answer the following:

(i) Define sub-Boolean algebra. State the necessary and sufficient condition for a subset becomes sub-algebra. Find all sub Boolean algebra of $\langle S_{110},\ D\rangle$. **05**

(ii) Prove the following Boolean identities: **02**

(1) $(x'\oplus y)*(x\oplus y)=y$ (2) $(x\oplus y\oplus z)*(y\oplus z)=(y\oplus z)$

(b) Use the Quine-Mccluskey algorithm to find the prime implicants and also obtain a minimal expression for function: $f(a,b,c,d)=\sum(15,14,13,6,5,2,1)$. **07**

OR

Q.3 **(a)** Use Karnaugh map to find a minimal sum-of-product expression for the function given by $\sum(0,1,2,3,6,7,13,14)$ in four variables w, x, y, z. **07**

(b) Answer the following:

(i) Given an expression $\alpha(a,b,c,d)=\sum(2,3,6,8,12,15)$, determine the value of $\alpha(3,5,10,30)$ where $3,5,10,30 \in \langle S_{30}, D\rangle$. **04**

(ii) Find the sum of products expansions of Boolean functions $f(x,y,z)=(x+z)y$ **03**

Q.4 **(a)** Define group homomorphism; prove that group homomorphism preserves identities, inverses and subgroups. **07**

(b) Define cyclic group. Find generators of $\langle Z_{12}, +_{12}\rangle$. Also find its all subgroups. Which subgroups are isomorphic to $\langle Z_4, +_4\rangle$? Justify. **07**

OR

Q.4 **(a)** Show that if every element in a group is its own inverse, then the group must be abelian. Is the converse true? Justify. **07**

(b) Define symmetric group $\langle S_3, \lozenge\rangle$. Write its composition table. Determine all the proper subgroups of $\langle S_3, \lozenge\rangle$. Which subgroup is normal subgroup? Support your answer with reason. **07**

Q.5 **(a)** Define isomorphic graphs. Determine whether the digraphs G and H given in figure – 1 (a), (b) are isomorphic. **07**

(b) Define node base of a digraph. Find all node base of the digraph shown in figure – 2. List out all the properties of a node base. Explain why no node in node base is reachable from any other node in node base. **07**

OR

Q.5 **(a)** Define: path, simple path, elementary path. For the graph given in Figure – 3: **07**

(i) Find an elementary path of length 2 from v_1 to v_3.

(ii) Find a simple path from v_1 to v_3, which is not elementary.

(iii) Find all possible paths from node v_2 to v_4 and how many of them are simple and elementary?

(b) Define a directed tree. Draw the graph of the tree represented by $(A(B(C(D)(E))(F(G)(H)(J)))(K(L)(M)(N(P)(Q(R)))))$ Obtain the binary tree corresponding to it. **07**

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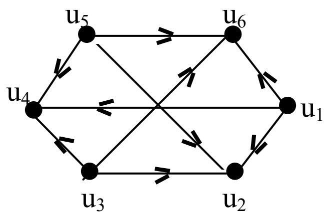
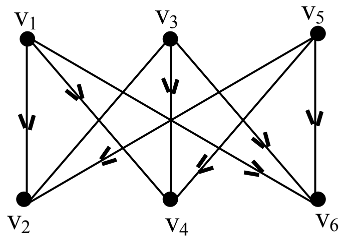


Graph G:
Figure – 1 (a)



Graph H:
Figure – 1 (b)

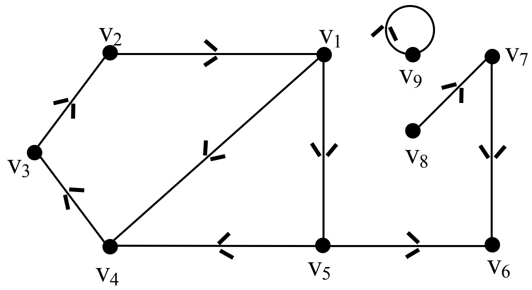


Figure – 2

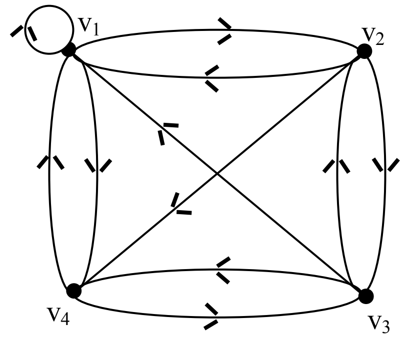


Figure – 3

Seat No.: ________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA- I[st] SEMESTER–EXAMINATION – MAY/JUNE - 2012

Subject code: 2610003 **Date: 31/05/2012**

Subject Name: Discrete Mathematics for Computer Science (DMCS)

Time: 02:30 pm – 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define: **07**

i) Join irreducible elements.
ii) Atoms of a Boolean algebra.

Determine Join-irreducible elements and atoms of following Boolean algebra.

i) (S_{210}, D)

ii) ⟨ P(S), ∩,∪, ', Φ, S⟩ where S = {a, b, c}

Also draw the Hasse Diagram.

(b) Define Lower bound and greatest lower bound. Let P = < 3, 5, 9, 15, 24, 45}, D> be a poset. Draw the Hasse diagram. Find **07**

i) the maximal element. & minimal element.

ii) The greatest and least element.

iii) the lower bounds of {3, 5}, if any & the upper bound of { 9, 15 } , if any

iv) GLB of {15, 45} & LUB of {3, 9, 15}.

Q.2 **(a)** State the importance & purpose of Discrete Mathematical Structures with its application to computers science. **07**

(b) **02**

i) Let P(x) be the statement " $x = x^2$ ".If the domain consists of the integers, what are the truth values of ∀x P(x)and ∃ x P(x)

05

ii) Define: Logical Equivalence of the statement formula. Without constructing truth table show that (˥p ∧ (˥q ∧ r)) v (q ∧ r) v(p ∧ r) ≡ r

OR

(b) i) Define: Disjoint sets. If A_1 ={{1, 2}.{3}}, A_2 = {{1},{2,3}} and A_3= {{1, 2, 3}}, then show that A1, A2, and A3 are mutually disjoint. **03**

ii) Define law of Modus Ponen and Law of Hypothetical Syllogism with an example. **04**

Q.3 **(a)** i) Define: Equivalence relation. If I be the set of integers and if R be defined **04**

by " a R b iff a – b is an even integer" where a, b ϵ I, then show that the relation R is an equivalence relation.

ii) Define giving example for each term **03**

1. Sublattice

2. Complemented lattice

3. Modular lattice

(b) i) Define: Maximal Compatibility Block. Let the compatibility relation on a set {1, 2, 3, 4, 5, 6} be given by following matrix. Construct the graph and find the maximum compatibility blocks **04**

2	1				
3	1	1			
4	1	1	1		
5	0	1	0	0	
6	0	0	1	0	1
	1	2	3	4	5

ii) State the absorption law for lattice. Verify it for (S_{45}, D) by taking any two elements. **03**

OR

Q.3 **(a)** i) Find the value of Boolean Expression. **04**

$\alpha(x_1, x_2, x_3, x_4) = [x_1 * (x_2 \oplus x_1') * (x_3 * x_4' * x_2')] \oplus (x_1 * x_4)$ where

$x_1 = 5, x_2 = 6, x_3 = 15, x_4 = 3$ in Boolean algebra $<S_{30}, gcd, lcm, ‘, 30>$ and $n' = 30/n$.

.ii) Prove the Boolean identities **03**

a) $(a * b) \oplus (a * b') = a$

b) $a * (a' \oplus b) = a * b$

(b) i) Use the Quine-Mccluskey algorithm to find the prime implicants and also obtain a minimal expression for function: $f(a,b,c,d)=\Sigma(1,2,5,6, 13, 14, 15)$ **04**

ii) Obtain the sum of product canonical form of Boolean expression in three variables x_1, x_2, x_3 for $(x_1 \oplus x_2) * x_3$ **03**

Q.4 **(a)** Define: Group and Abelian group. Show that in a group (G, ∗), if for a, b ϵ G, $(a * b)^2 = a^2 * b^2$, then (G, ∗) is an Abelian group. Prove that the set {1, -1, i, -i} form an Abelian multiplicative group (G, x) where i is an imaginary no. $i = \sqrt{-1}$. **07**

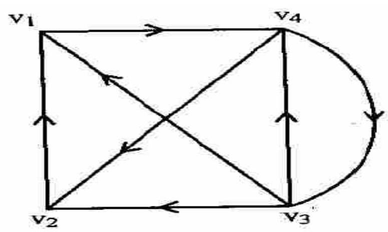
(b) Define: Group Homomorphism, Group Isomorphism and Kernel of the homomorphism. Prove that G: $(Z_4 , +4) \longrightarrow (Z_5^*, x_5)$ is isomorphism. **07**

OR

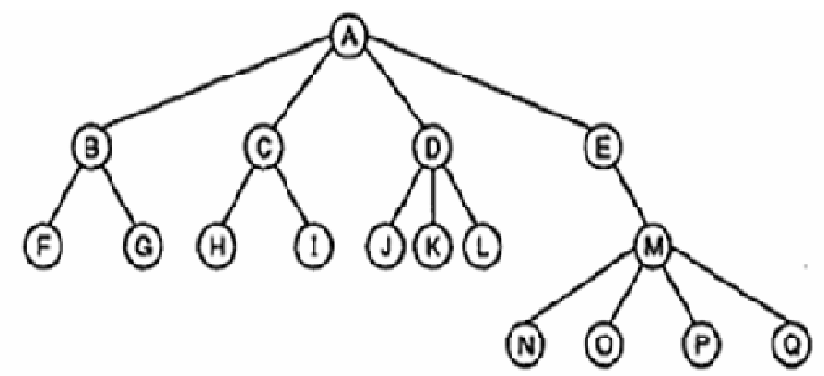
Q.4 **(a)** Define Subgroup of a group Find all subgroups of cyclic group of order 12 with generator 'a'. Also find order of generators of G. **07**

(b) Define symmetric group $(S_3, \Diamond)$. Write composition table of all permutations defined on the symbols 1, 2, & 3 Determine all the proper subgroups of $(S_3, \Diamond)$. Which subgroup is normal subgroup? **07**

Q.5 **(a)** Define adjacency matrix of a graph and obtain the adjacency matrix (A) for the following graph. What do transpose of adjacency matrix (A^T) indicate? Draw its graph. State the indegree and outdegree of all the vertices. Find A^2 and interpret in detail by stating the results. **07**

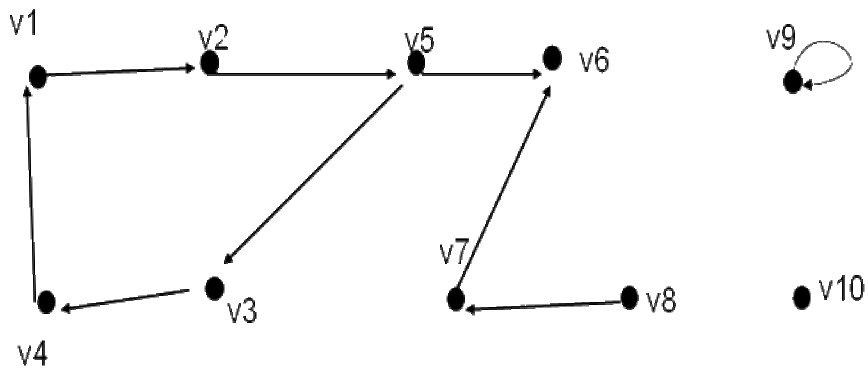


(b) i) Define Forest with an example **02**
ii) Define Binary tree. Convert the given tree into the Binary tree. **05**



OR

Q.5 **(a)** Define node base of a diagraph. State its properties. Find all node base of the diagraph given below: **07**



(b) Define rooted tree, level of a vertex, leaf, descendents and ancestor of a vertex with a suitable example. Prove that a full m-ary tree with i internal vertex has n = mi +1 vertices **07**

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Gujarat Technological University
Master of Computer Applications
Semester-I

Subject Name: **Discrete Mathematics for Computer Science (DMCS)**
Subject Code: **2610003**

Objectives: The objective of this course is to present the foundations of many basic computer related concepts and provide a coherent development to the students for the courses like Fundamentals of Computer Organization, RDBMS, Data Structures, Analysis of Algorithms, Theory of Computation ,Cryptography, Artificial Intelligence and others. This course will enhance the student's ability to think logically and mathematically.

Prerequisites: Knowledge of basic concepts on Sets, different operations on sets, binary operations, functions.

Contents:

1. **Introduction and Pre-requisites** **[6 Lectures]**
   Importance & Purpose of Discrete Mathematical Structures; Applications; Set Theory, Functions, Relations, etc.

2. **Mathematical Logic:** **[8 Lectures]**
   Introduction, Connectives, statement formulas, principle of substitution, validity of arguments, Quantifiers, Proof techniques.

3. **Lattices and Boolean Algebra:** **[10 Lectures]**
   Relation and ordering, partially ordered sets, Lattices as poset, properties of lattices, Lattices as algebraic systems, sub-lattices, direct product and homomorphism, complete lattices, bounds of lattices, distributive lattice, complemented lattices.
   Introduction, definition and important properties of Boolean Algebra, Sub Boolean algebra, direct product and homomorphism, join-irreducible, meet-irreducible, atoms, anti atoms, Stone's representation theorem. (Without Proof),

   Note: No proof is required for Theorems or Results on lattices and Boolean Algebra. Theorems should be justified and explained by suitable examples.

4. **Applications of Boolean Algebra :** **[10 Lectures]**
   Boolean expressions and their equivalence, Minterms and Maxterms, Free Boolean algebra, Values of Boolean expression, canonical forms, Boolean functions, representation of Boolean function, Karnaugh maps, minimization of Boolean function, Quine-Mccluskey algorithm, Application to Relational Database.

5. Group Theory : [6 Lectures]

Definition and examples of groups, abelian group, cyclic groups, permutation groups, subgroups & Homomorphism, Cosets and Lagrange's Theorem (without proof), Normal subgroups, Quotient Groups.

6. Graph Theory: [8 Lectures]

Basic concepts of Graph theory, paths, reachability and connectedness, matrix representation of graph, trees.

Text Books:

1. "Discrete Mathematical Structures with Applications to Computer Science", J. P. Tremblay and R.Manohar ,Tata McGraw-Hill
2. "Discrete Mathematical Structure", D. S. Malik, M. K. Sen, Cengage Learning

Reference Books:

1. Discrete Mathematics and its applications, Tata McGraw-Hill, 6^{th} edition, K. H. Rosen.
2. Discrete Mathematical Structure, Pearson Education, Bernard Kolmann& others, Sixth Edition
3. Discrete Mathematics with Graph Theory, PHI, Edgar G. Goodaire, Michael M. Parmenter.
4. Logic and Discrete Mathematics, Pearson Education, J. P. Tremblay and W. K. Grassman.

Chapter wise coverage from the Text Books:

1. From Book # 1
   Chapter – 2, article 2-3 (2-3.1 to 2-3.9)
   Chapter-3, article 3-5 (3-5.1 to 3-5.4) up to Theorem 3-5.8
   Chapter – 4, articles 4-1 to 4-4
   Chapter – 5, article 5-1 (5-1.1 to 5-1.4)
2. From Book # 2
   Chapter – 1, articles 1.2 to 1.5
   Chapter-3 article 3.3

Accomplishment of the student after completing the course :

The student will be able to apply concepts to RDBMS, perform minimization of Boolean functions, shall learn the fundamentals representations methods of graphs and trees. They shall be able to use different logical reasoning to prove theorems.